

**Exercise 29**

If  $H(\theta) = \theta \sin \theta$ , find  $H'(\theta)$  and  $H''(\theta)$ .

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**Solution**

Calculate the first derivative using the product rule.

$$\begin{aligned} H'(\theta) &= \frac{d}{d\theta}[H(\theta)] \\ &= \frac{d}{d\theta}(\theta \sin \theta) \\ &= \left[ \frac{d}{d\theta}(\theta) \right] \sin \theta + \theta \left[ \frac{d}{d\theta}(\sin \theta) \right] \\ &= (1) \sin \theta + \theta(\cos \theta) \\ &= \sin \theta + \theta \cos \theta \end{aligned}$$

Calculate the second derivative using the product rule again.

$$\begin{aligned} H''(\theta) &= \frac{d}{d\theta}[H'(\theta)] \\ &= \frac{d}{d\theta}(\sin \theta + \theta \cos \theta) \\ &= \frac{d}{d\theta}(\sin \theta) + \frac{d}{d\theta}(\theta \cos \theta) \\ &= (\cos \theta) + \left[ \frac{d}{d\theta}(\theta) \right] \cos \theta + \theta \left[ \frac{d}{d\theta}(\cos \theta) \right] \\ &= (\cos \theta) + (1) \cos \theta + \theta(-\sin \theta) \\ &= 2 \cos \theta - \theta \sin \theta \end{aligned}$$